Dark flight calculations: how accurate can they be?

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Dark flight calculations rely on accuracy of input data. The resulting uncertainties are analyzed and illustrated on an example of a simulated meteorite fall. It turns out that the biggest problem is uncertainty in the deceleration of the incoming body, together with meteorological data about wind velocity (speed and direction). The expected uncertainty in the calculated coordinates of the impact point defines a probability ellipse which is highly stretched in the direction of the average wind direction, with a semi-major axis of a few kilometers, and a semi-minor axis of a few hundred meters in size.

1 Introduction

Dark flight equations are well known (Ceplecha, 1987) and there are no large obstacles on the theoretical side of the problem, as long as the problem is limited to a single falling body. Dark flight calculations that include body disintegration are quite a different story (see, e.g., Barri, 2010) and will not be considered here. Additionally, effects of the body rotation are ignored, as are effects of the rotation of the Earth. The reason is that these effects are much smaller than the uncertainties produced by other input variables.

From the practical side of the problem, most parameters that enter the equations are not known with a sufficient precision, resulting in uncertainties in the body path through the atmosphere and location of the impact point on the ground. In this paper, the results of a simple analysis of the influence of individual parameters on the location of the impact point are discussed. At the end, a Monte Carlo simulation of the combined effects of all uncertainties together is performed, based on a hypothetical small meteorite that enters the atmosphere somewhere at a latitude of about 45° N.

To start with, let us assume that a camera network (e.g., the Croatian Meteor Network, or CMN for short) records a bright meteor and provides good triangulation data of its flight through the upper atmosphere. The raw triangulation will provide the direction of the meteor flight and coordinates of a set of points on the meteor trail, separated by a fixed time interval, usually the time between two successive frames of the recording camera (in the case of a camera network, like CMN). As the frame rate in modern cameras is set by a quartz oscillator, it can be taken as absolutely accurate for our purpose. From this set of data, additional parameters are derived needed for the dark flight calculations.

For a single well-defined point on the bright part of the meteor path, usually near or at the very end of the recorded path, the minimum data set is composed of the following data:

1. the coordinates of the point, given as geographic or cartesian coordinates, plus the height above the geoid;
2. the velocity vector (speed and direction of the flight) in the same coordinate system; and
3. the deceleration.

The coordinates of the starting point are usually accurate enough for the calculations. Additionally, any error in the starting point coordinates simply shifts the whole meteorite path by the same amount. However, the velocity has to be determined from several adjacent path points by some numerical procedure that is effectively a numerical derivation (we have to divide the distance between two points by the time of flight between them). Any errors in point positions are enlarged manyfold during such a calculation, an effect that is well known in numerical mathematics. Usually, some sort of smoothing is performed on the points first, to reduce the uncertainty in velocity.

The uncertainty is additionally magnified in calculations of the deceleration, as deceleration is a derivative of the velocity. Also, the deceleration changes rapidly, and can be quite small in the part of the trail that is recorded without saturation, which often spoils the astrometric accuracy in the last part of bright meteor trails.

This data set is then used to initiate calculations of the dark flight using the drag equations. They require several additional parameters. First of all, we consider the *resistance coefficient*, which will be referred to as GS in this article. This coefficient corresponds to the ratio of friction force and inertial force of the flying body and is defined as (Ceplecha, 1987)

$$GS = \frac{C_v A}{m},$$

where C_v is the drag coefficient, A the reference area, and m the mass of the body under consideration. The drag coefficient C_v is a dimensionless parameter that depends on the shape and surface roughness of the body, and on its velocity. The reference area A is usually defined as the area of the cross section of the object on a plane perpendicular to the direction of fight.

The parameter GS can be determined from the triangulation data for a chosen point of the meteor trail as

$$GS = \frac{a}{\rho_A v^2},$$

where a is the deceleration at this point (in absolute value), ρ_A the atmospheric density, and v the speed of the meteor. A closer look at this equation reveals that the uncertainty in GS is larger than the uncertainty in deceleration, as the latter is divided by the square of the speed, thus magnifying the effect of uncertainty in speed and adding it to the uncertainty in deceleration. We also need to know the atmospheric density at the appropriate height, usually several tens of kilometers above the ground level.

Note that by using this procedure we cannot separate individual factors from which GS is composed. We obtain only the GS value from the triangulation data. The calculations are mass independent, i.e., they do not depend on the mass of the moving body, only on the GS value. Regardless of their mass, all bodies with the same value of the resistance coefficient GS will follow the same path through the atmosphere.

To estimate the mass of the falling body, we have to guess its shape and surface roughness (to obtain the appropriate drag coefficient) and density (to infer the ratio of mass and cross-sectional area). This procedure, under the assumption of a non-rotating spherical body with a rough surface leads to the so called *dynamic mass*, which gives us a rough estimate of the real mass of the falling body (errors of a few times to an order of magnitude can be expected).

As the drag coefficient is speed-dependent, and as our body slows down from a large supersonic speed to almost a standstill, we have to take the variations of the drag coefficient into account. Luckily, we only need to know relative changes of the drag coefficient and change the resistance coefficient GS accordingly. Several authors provide data about the drag coefficient variations that we can use (Ceplecha, 1987; Carter et al., 2009). At large velocities, the drag coefficient is fairly constant, and surface-related differences at small velocities do not have a large influence on the final result as the body near the ground is falling almost vertically.

To summarize, the resistance coefficient requires us to provide some information about the drag coefficient of the body and the atmospheric density. Additionally, the drag equations themselves require the wind velocity vector (velocity and direction) to be known. Thus, we need the following set of additional data:

1. the relative variation of the drag coefficient as a function of speed;
2. the atmospheric density as a function of height, from the ground up to the trail point we use for the start of calculations, usually at the height of several tens of kilometers; and
3. the wind velocity (speed and direction) in the same height range.

2 Discussion

From the above elaboration, it follows that there are several input parameters whose uncertainties influence the results of dark flight calculations.

To illustrate the impact of each of them, a simple procedure is used. First, a dark flight calculation for an ideal case (everything known) is done, and the coordinates of the impact point on the ground are determined. Then, a set of calculations is done in which one parameter is changed by an amount equal to the expected uncertainty in its value, while all other parameters are kept the same, and the results are compared.

This procedure gives a clear image of the importance of the accuracy of each single parameter, but cannot provide information on their combined effect. Thus, an additional calculation is performed in which all parameters are simultaneously changed by a randomly chosen amount within the limits of the expected uncertainty for each parameter. This last calculation is repeated many times, providing a distribution of expected impact points on the ground. Such a procedure is called a Monte Carlo simulation.

As a starting point, data for one bright bolide with a good triangulation accuracy observed by CMN are taken. Data is given in Table 1, together with the impact of uncertainties of each of them on the position of the ground impact point of the falling body, determined by the above procedure.

To simplify, the calculations are done in the cartesian x-y-z coordinate system that was used by Ceplecha, in which the z-axis is the height, the x-axis points into the initial direction of the meteor flight and the y-axis completes the right-handed coordinate system. Thus, the xy plane is the horizontal plane, and the positive y-axis is to the left of the x-axis if we look in the direction of meteor flight. The starting point is set at $x = 0$, $y = 0$, and z the initial height.

Performing the initial calculation with GS being kept constant, the atmospheric density taken from the standard atmosphere model and wind ignored, we obtain that the impact point is about 3.5 km away from the last observed point, in the direction of the flight (Figure 1). We took data for the air density from the standard US atmosphere 1976 (NOAA et al., 1976), but this is not critical as all current standard atmospheric models are practically identical for heights under 100 km that are of interest to us.

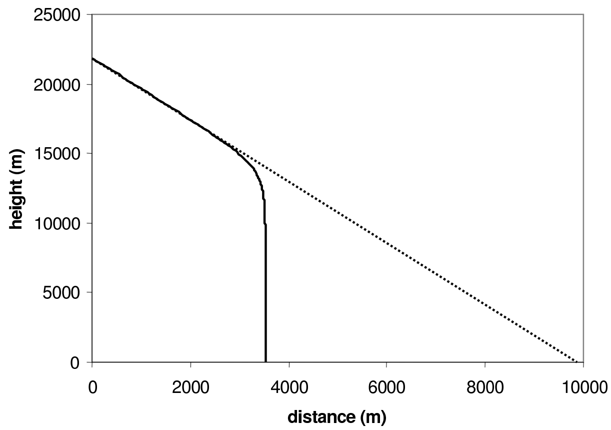

Figure 1 – The full line shows the calculated meteorite path in an ideal atmosphere with no wind, and, just for curiosity, the dotted line shows the path it would have in vacuum.

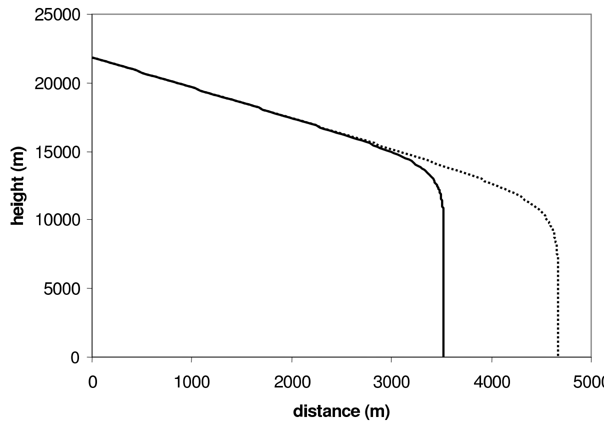

Figure 2 – The full line is the calculated meteorite path in an ideal atmosphere with no wind, and the dotted line is the path the body would travel if the initial deceleration is 50% larger. The resulting change in the position of the impact point is about 1400 m.

2.1 Triangulation data

The uncertainty in the location of the impact point due to the uncertainties in the triangulation data were analyzed first. The uncertainty in the position of the initial point simply moves the impact point by the same amount, in our example by ±100 m in the x and y coordinates. The uncertainties in other triangulation data are summarized in Table 1.

Table 1 – Data for the initial calculating point, together with typical CMN uncertainties of triangulation data and their impact on the position of the ground impact point of the supposed meteorite.

Parameter	Uncertainty	On ground
Height	22000 ± 100 m	very small
Position (xy plane)	0 ± 100 m	± 100 m
Speed	5000 ± 500 m/s	± 500 m
Deceleration	5000 ± 2500 m/s^2	±1400 m
Azimuth of track	$0.^\circ0 \pm 0.^\circ5$	± 300 m
Impact angle	$67.^\circ0 \pm 0.^\circ5$	± 100 m

It is obvious that the biggest problem in this data set is the very large uncertainty in deceleration. This is additionally illustrated in Figure 2. On the other hand, it should be stressed that the uncertainty in the position of the impact point affects only the x-direction, because the drag force is always opposite to the direction of the flight.

2.2 Drag coefficient

The drag coefficient enters the *GS* parameter in the drag equations. Assuming that the mass of the body is constant, which in our case means that the ablation ended and no disruption takes place after the initial point, any change of the drag coefficient will cause the same relative change of *GS*. The assumption that the body does not change its orientation during the flight is also involved, as changing the orientation will change the cross-sectional area that is also a part of the expression for *GS*, and, even worse, can drastically change the drag coefficient itself.

In reality, from the data available from video networks of any kind, we cannot conclude if any of these two assumptions are valid or not. For simplicity, we thus assume they are.

The drag coefficient itself is a complicated function of the object shape, surface roughness, and speed. Again, we do not know anything about the body shape and surface properties. From known meteorites, we see that they can have many different shapes, although a large portion of them can be approximated by a sphere or an ellipsoid. Their surface is usually rough, sometimes even very rough. Relying on these facts, we assume that the flying body has a spherical shape and a rough surface. This helps us insofar that, for a sphere, there exist a lot of experimental and theoretical data about the drag coefficient. The dependence of the drag coefficient on shape and surface conditions of the body is illustrated in Table 2. We see that, even for a sphere, the uncertainty due to the unknown surface roughness can be 50% or more. Luckily, as was said before, we do not have to know the absolute value of the drag coefficient, just its dependence on the speed.

Table 2 – Drag coefficient for simple bodies at very large speeds.

Body	Drag coefficient
Smooth sphere	0.8
Rough sphere	≈ 1.6
Very rough sphere	≈ 1.2
Smooth ellipsoid	≈ 1.0
Hemisphere	≈ 1.6
Flat disk	≈ 5

The problem that remains is the speed dependence of the drag coefficient. The meteorite enters the atmosphere with a very large speed and slows down. At the initial point of our calculations, it still flies with the speed of several tens of Machs. At very large speeds, the drag coefficient tends to be constant, but, at small speeds, it varies considerably. The newest compilation of data about the speed dependence of the meteorite drag coefficient is given by Carter et al. (2009), but the data provided by Ceplecha (1987) are quite close. Our calculations show that the total uncertainty due to the drag coefficient is about 300–500 m, again in the direction of flight.

2.3 Atmospheric density

The atmospheric density profile can be obtained from the meteorological measurements that are usually available on web pages of major meteorological institutions. Such meteorological data are usually gathered by atmospheric sounding techniques, i.e., by launching a meteorological balloon that carries a standard set of meteorological instruments (Strangeways, 2003). Such measurements are done twice a day at selected meteorological stations distributed around the world. However, the stations themselves are separated by several hundreds to several thousands of kilometers, so it is difficult to infer the appropriate data for the impact point if it is far away from a sounding station. The same problem occurs if the impact time is several hours away from the time of the nearest measurement.

If we do not have data about the atmospheric density profile, the density profile is taken from the so called "standard atmosphere" model that represents the average atmospheric conditions. The standard US atmosphere 1976 (NOAA et al., 1976) is used here, but, for heights below 100 km, other currently used atmospheric models provide the same or very similar density profiles. For instance, Ceplecha (1987) used the CIRA 1972 atmospheric model, which provides the identical formula for the atmospheric density as the US 1976 atmospheric model. This formula is used for density estimation in calculating the atmospheric path shown in Figure 1.

Actual atmospheric conditions can be quite different from the average situation represented by a standard model. Thus, several measurements of atmospheric conditions from the meteorological station in Zagreb, Croatia (MHSRH, 2011) were randomly chosen among night measurements to provide data about actual density and wind profile (speed and direction that are used later on in this article) to see what results can be expected from the variations of atmospheric density. To avoid the need for interpolating the data, it is assumed that our hypothetical meteorite enters the atmosphere directly above the sounding station at the moment of the sounding. Calculations performed with actual data show that different density profiles can shift the impact point by some 200 m at most, relative to the position obtained by using an idealized density profile from a standard atmospheric model.

2.4 Wind speed and direction

Measured wind data show that in our region the wind profile can be very complex, with strong winds of different directions at different altitudes. The shifts obtained by repeating calculations with different actual wind profiles are huge, up to 2 km in any direction relative to the meteorite flight direction. However, on a particular night, the shifts are more consistent, and the uncertainty in the impact point coordinates is about 300 m, relative to the most probable impact point calculated with the best wind data for the time of the impact. An example of such a calculation is given in Figure 3. Note that we now have a shift in both horizontal coordinates, and that the path through the atmosphere can be very complicated. Actually, soon after the maximal deceleration, the body assumes free fall with the terminal velocity in vertical direction, and is moved around by the wind very effectively, so the horizontal velocity of the body is very close to the wind velocity.

A final remark has to be given here. Meteorologists neglect the vertical wind component, assuming that it is always very small. This is true for most of the time, but not always. Sounding data do not provide any clue about this wind component, so we cannot check it for sure, but in the author's opinion, the effects of any vertical wind component are much smaller than the other uncertainties already discussed, so this topic is not pursued further.

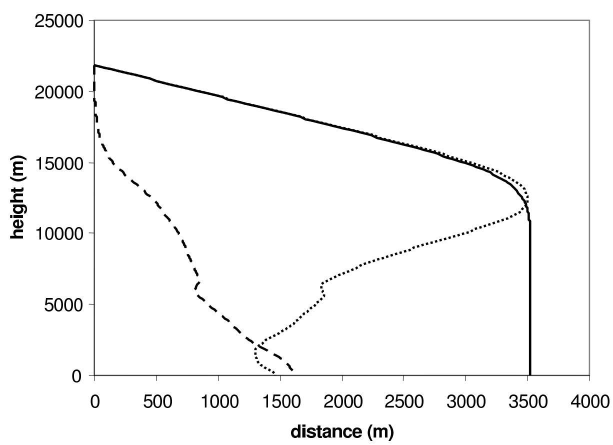

Figure 3 – The full line shows the calculated meteorite path in an ideal atmosphere with no wind, the dotted line shows the x-component of the path of the body moved by the wind, and the dashed line shows its y-component. Note that both shifts are very large in this example: about 2000 m in the x-direction and about 1600 m in the y-direction.

3 Mixing it all together: a Monte Carlo simulation

The previous analysis clearly shows the relative importance of uncertainties in input parameters involved in dark-flight calculations. However, it does not reveal the simultaneous effects of them all, because the effects of each of them are not all in the same direction. Some parameters change only the x-coordinate of the impact point, i.e., they "work" in the direction of the meteorite flight, or in the opposite direction, while other parameters affect both the x- and y-coordinates, i.e., they also "work" in the direction perpendicular to the flight direction. In such a situation, one of the solutions to the problem is to use the Monte Carlo simulation. In such a procedure, one produces an input data set in which all parameters are randomly changed by an amount that is between minus and plus the expected uncertainty of the parameter under consideration. The dark flight calculation is done with this "spoiled" data set, and the resulting coordinates of the impact point are saved. The whole procedure is then repeated many times, each time using a new "spoiled" data set, and all results are put together. In the present case, they all fall inside an ellipse which we can designate as the "search ellipse," i.e., we expect that the real meteorite would fall somewhere inside this ellipse. What is remarkable is that the dimensions of such an ellipse are measured in kilometers (Fig. 4).

From the example in Figure 4, we see that the search ellipse is highly stretched. This reflects the fact that the uncertainty in deceleration is by far the largest uncertainty in the process, and it defines the semi-major axis of the ellipse, while the uncertainty in the wind data, the only that can affect the y-coordinate of the impact point, is several times smaller, thus defining the semi-minor axis of the search ellipse. In our example, this ellipse has a semi-major axis of about 2200 m, and a semi-minor axis of about 300 m.

It can also be seen that the whole ellipse is translated and rotated. This is the consequence of the average wind and deceleration uncertainties working together.

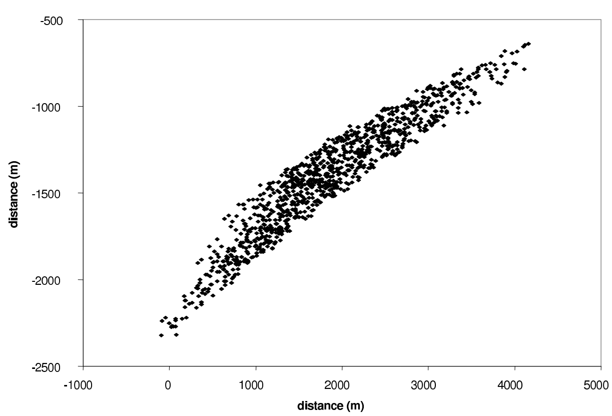

Figure 4 – The results of Monte Carlo simulations. One thousand virtual meteorites were sent through the atmosphere while all variables entering the dark flight calculations were simultaneously and randomly changed, as described in the text. Locations of the impact points of these meteorites define the possible fall area of the body.

4 Conclusions

It is clear from the previous analysis that there are many input variables whose uncertainties influence the results of a dark-flight calculation. From the triangulation side of the problem, the biggest problem is the inaccuracy in deceleration values which results in a quite large uncertainty in the position of the impact point, in the original direction of the flight of the incoming body. The uncertainty of the impact point reflects the uncertainty in the starting position of the body, and is in the order of hundreds of meters in the case of CMN. The inaccuracy in velocity (both in speed and in the direction of the flight) produces shifts on the ground that are a few times larger.

From the physical side of the problem, the uncertainty in drag coefficient causes an uncertainty of the order of one or two hundred meters, if the body has a fairly regular shape and flies in a stable orientation, which is usually the case.

The largest uncertainty is caused by meteorological conditions, where the wind profile is the main culprit, the effects of a varying atmospheric density being much smaller. The wind can shift the flying body a few kilometers in any direction, which we can call the direction of the average wind, as it is close to the average of wind speed and direction over the part of the atmosphere the dark flight is calculated over.

To take into account the effects of all uncertainties combined, a Monte Carlo simulation, or a similar analysis, has to be performed, resulting in an area on the ground within which we can expect the body to fall. This area is usually delimited by an ellipse, which in our case has a semi-major axis of about 2200 m, and a semi-minor axis of about 300 m.

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